

Salt Flux and Salinity of Growing Sea Ice



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Project was:

Computational fluid dynamics simulations of ice growth
(*science*)

Calibration of permeability–porosity relationship with field data
(*engineering*)

Development of a simple, explicit “model” (i.e. an equation)
to describe sea ice salinity as a function of growth conditions and
distance from the ice–ocean interface
(*engineering, unlike Andrew*)

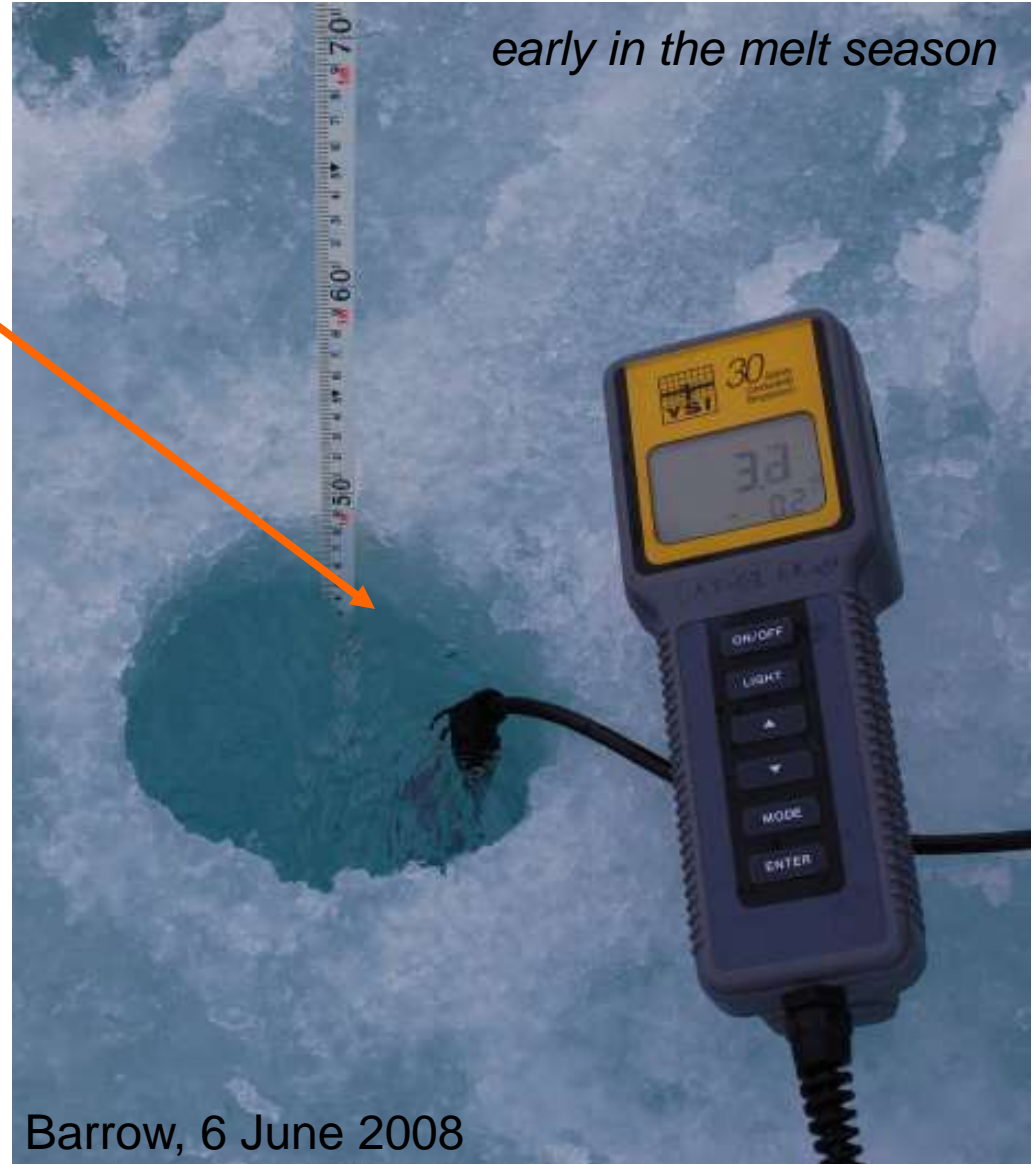
Calibrate and verify explicit model with CFD simulations and
experimental / field data
(*engineering*)

This talk: use only empirical fit to salinity vs. growth rate data

Example of (massive) lateral fluid flow



gap

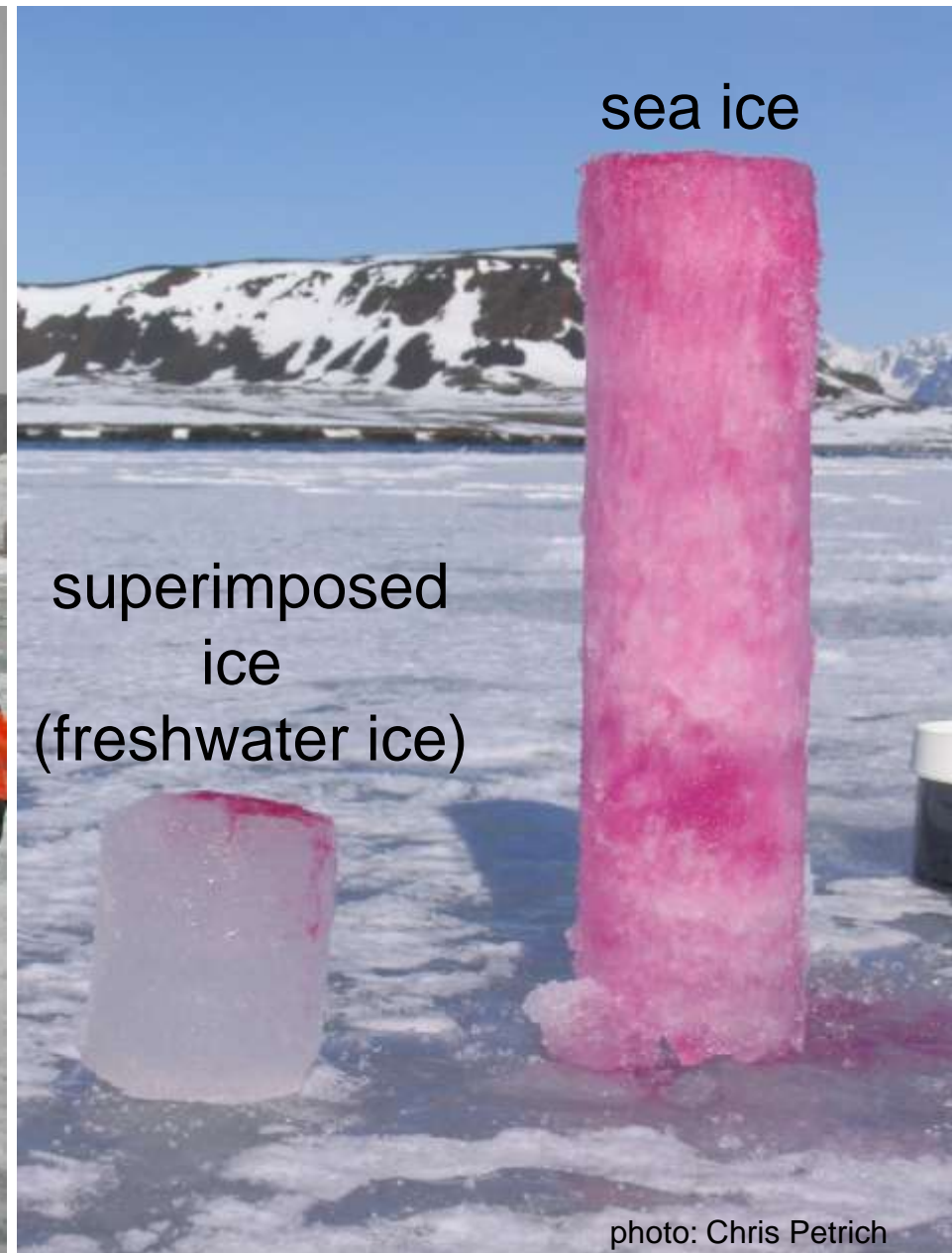


early in the melt season

Barrow, 6 June 2008

(show movie)

Sea ice can be quite permeable



In response to our discussion yesterday

Percolation theory: why a critical (“cut-off”) porosity of 0.05 of all porosities?

→ Importance for mushy layer theory: is the pore space connected,
i.e. is the difference between total brine volume and
connected (effective) brine volume significant?

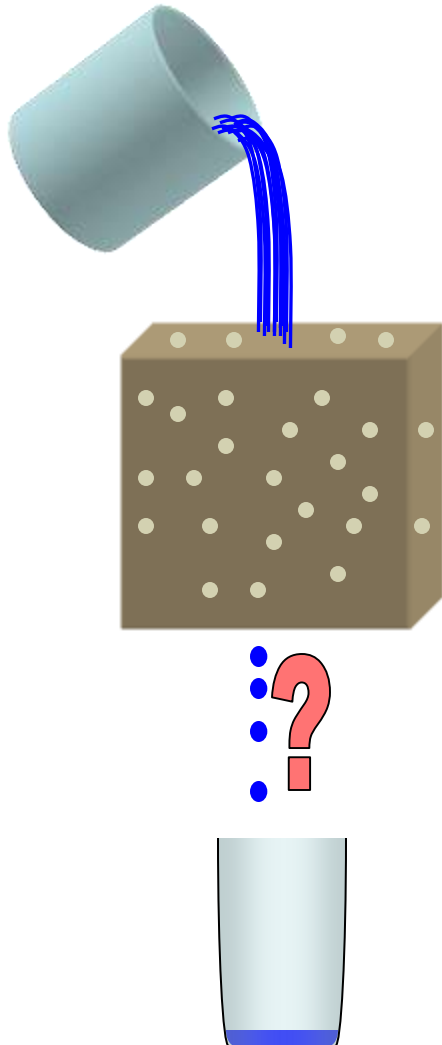
Rest of my presentation will be based on fluid dynamics modeling a la Danny,
i.e. assuming connected pore space and WITHOUT percolation threshold assumptions

Is there counter flow inside brine channels?

What is the brine salinity inside of brine channels,
in particular at the ice–ocean interface?

“Stable” bulk salinity (i.e. approx. 10 to 20 cm above the ice–ocean interface)

Fundamental question in percolation theory



something like:

$$\frac{\text{volume of pores}}{\text{volume of material}}$$

*Given the porosity of a material,
is the pore space sufficiently connected
to allow a fluid to percolate?*

this implies that there is a difference between

total porosity f_t and

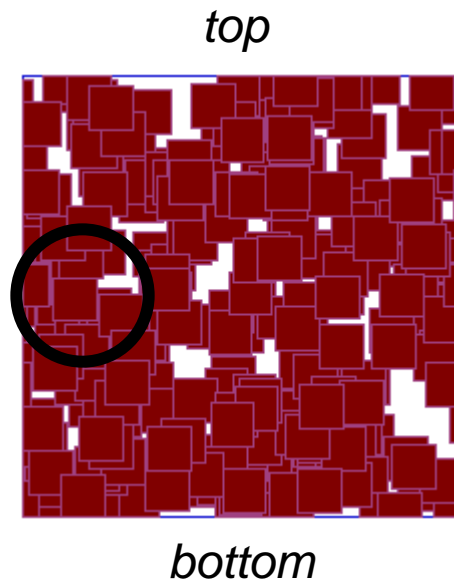
effective (connected) porosity f_e

we will consider random porous media of the simplest form

Percolation example

- 2-dimensional domain (10 x 10)
- periodic horizontally
- add pockets at random locations
- pocket size 1 x 1
- test for vertical percolation

$$\begin{aligned} N &= 300 \\ f_t &= 0.00 \\ f_e &= 0.50 \end{aligned}$$



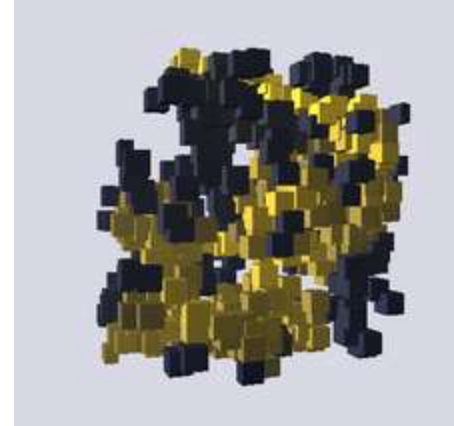
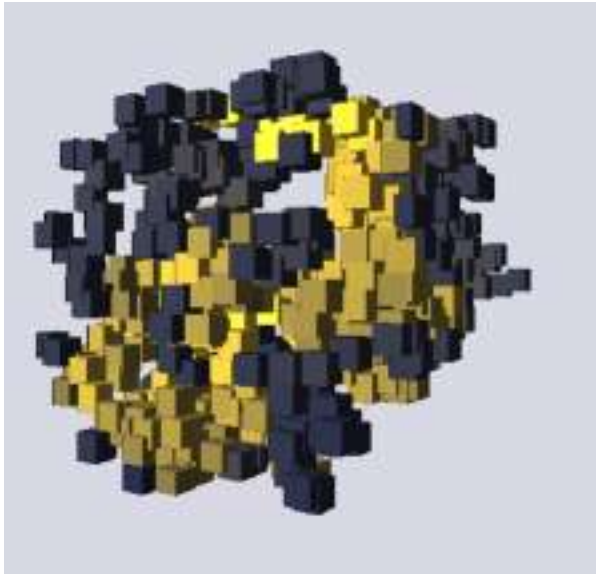
**cluster formation
and growth**

PERCOLATION

porous medium

Illustration of dead ends in 3D

percolating cluster at the percolation threshold



yellow: fluid flow

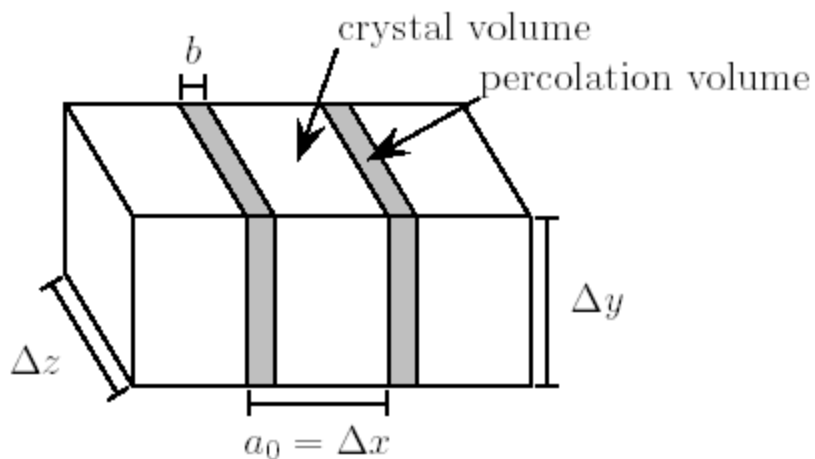
blue: no fluid flow (dead ends)

Monte Carlo percolation model

- square pockets are added at random into a large domain
- effective and total porosities are recorded

3 cases considered:

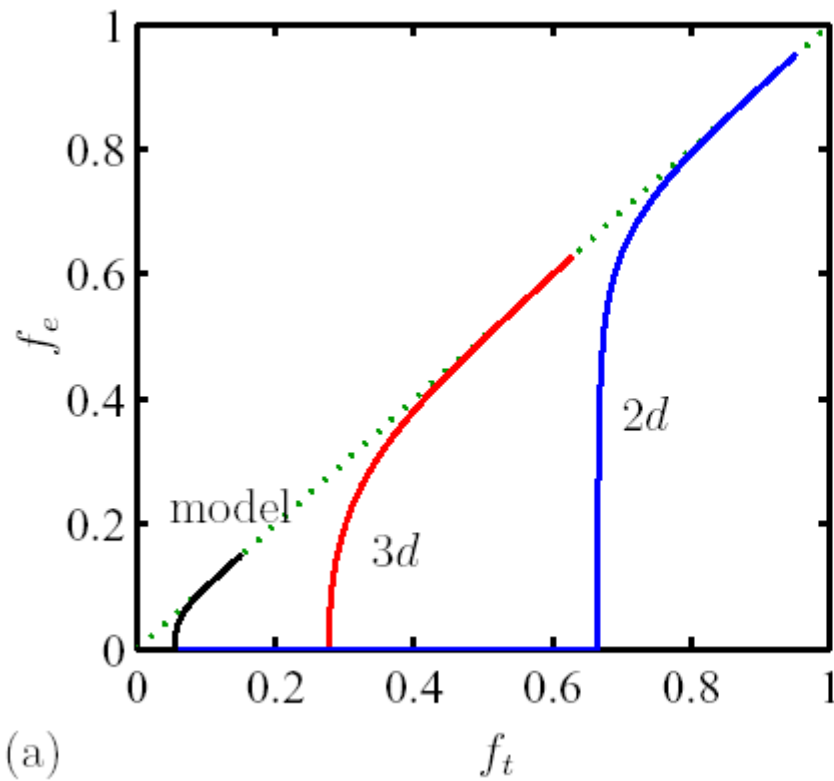
- 2D domain, square pockets
- 3D domain, cubical pockets
- 3D domain with **excluded volume** (e.g. ice crystals), cubical pockets



*pockets are placed
in crystal volume only
if they are attached to
an existing clusters*

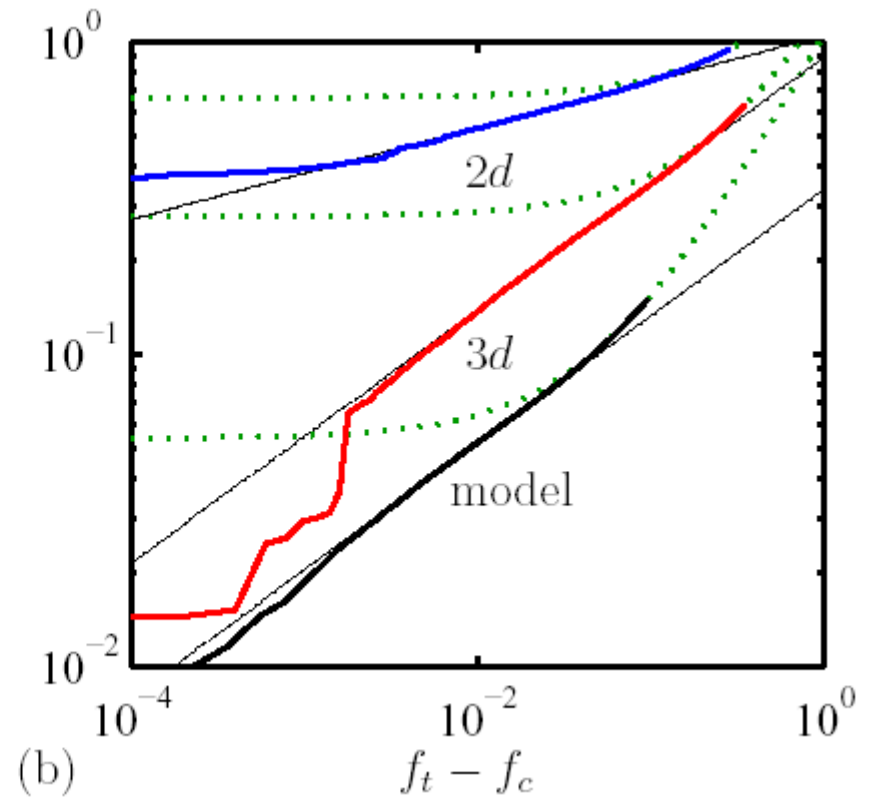
(Petrich et al., 2006)

Monte Carlo percolation model - results



(a)

$$f_e \rightarrow f_t \quad \text{for} \quad f_t \gg f_c$$



(b)

$$f_e \propto (f_t - f_c)^\beta \quad \text{for} \quad f_t \gtrsim f_c$$

Monte Carlo percolation model – approximation of results

$$f_e \approx \begin{cases} 0 & \text{for } f_t < f_c, \\ A(f_t - f_c)^\beta & \text{for } f_c \leq f_t \leq f_x, \\ f_t & \text{for } f_x < f_t \leq 1 \end{cases}$$

numerical results seem to justify the following [approximations](#):

- the relationship between f_e and f_t above is continuous at f_x
- the first derivative of the relationship between f_e and f_t above is continuous at f_x

thus:

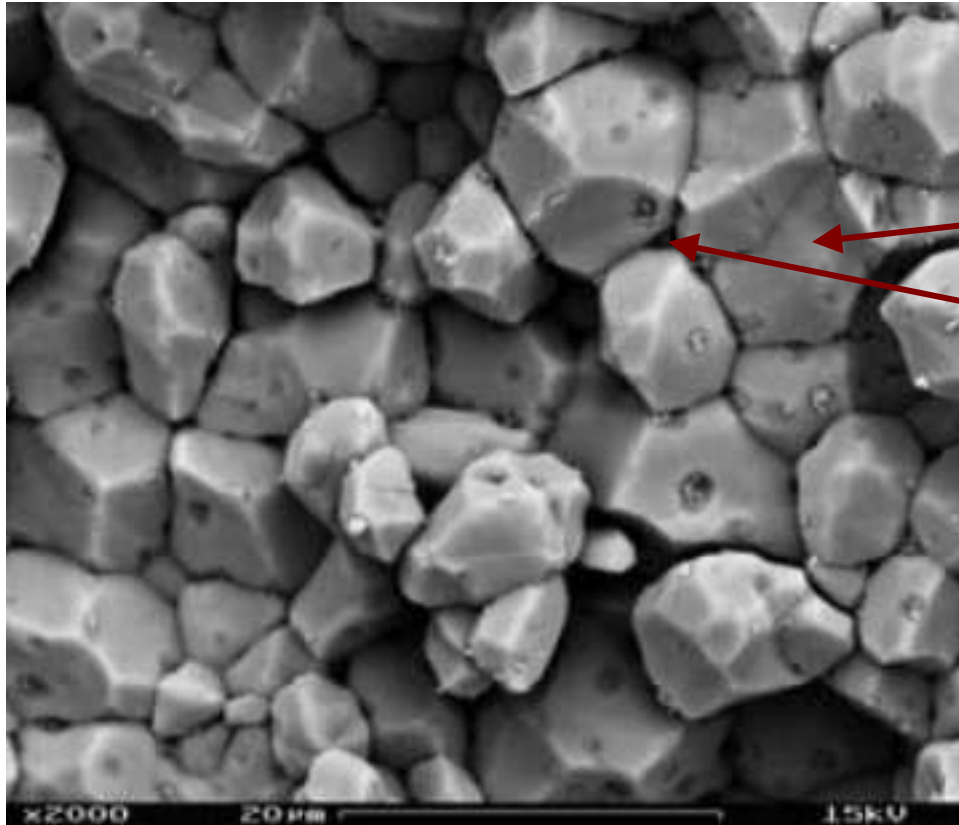
$$f_x = \frac{f_c}{1 - \beta}$$

$$A = \frac{1}{\beta} \left(f_c \frac{\beta}{1 - \beta} \right)^{1 - \beta}$$

hence: one needs to know only

- system dimension (3D beta=0.41)
- critical porosity

SEM image of compressed calcite aggregates

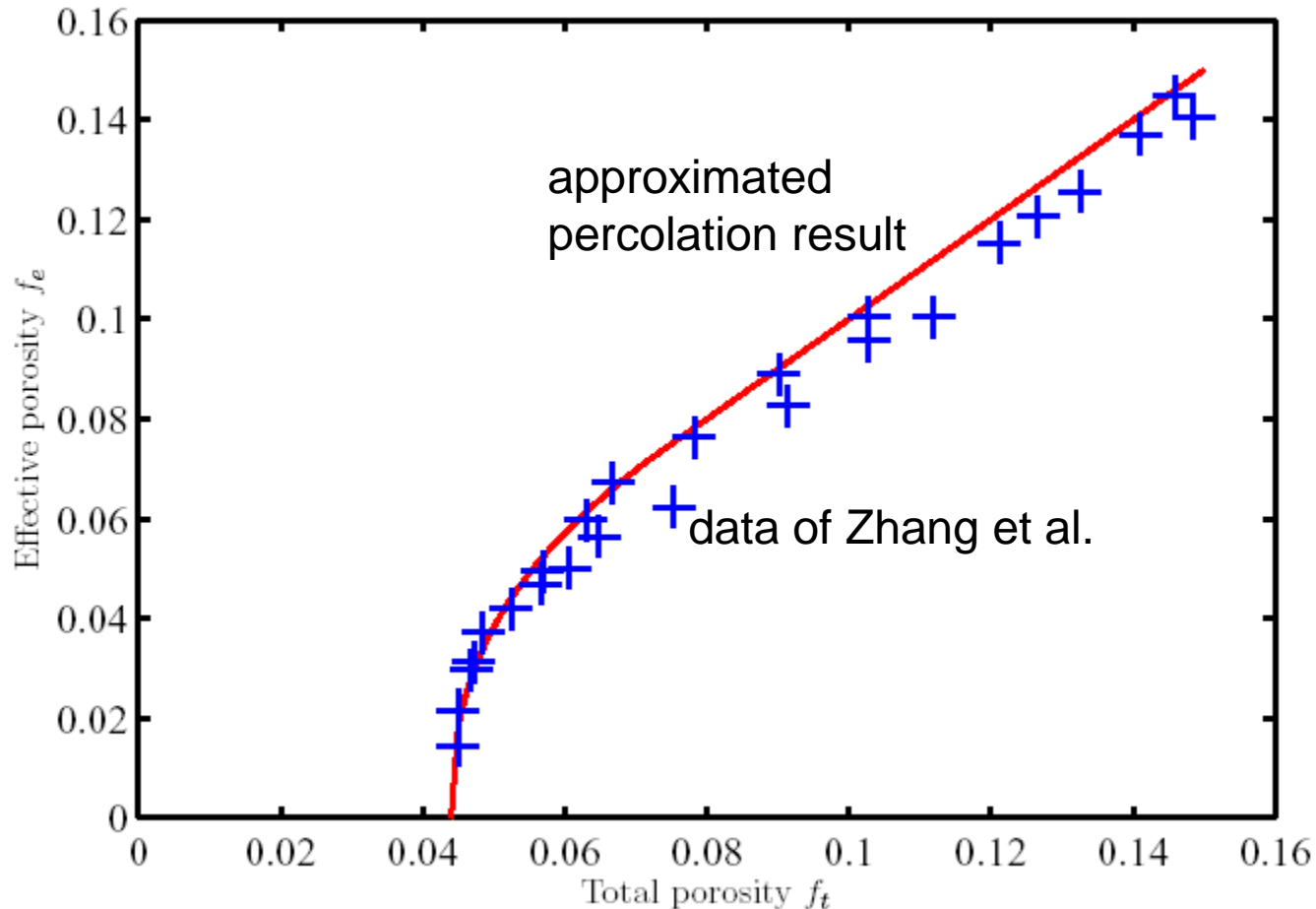


*crystals vs.
potentially
percolating volume
in grain boundaries
and pores*

Freund et al. (2001)

Percolation model – comparison with data

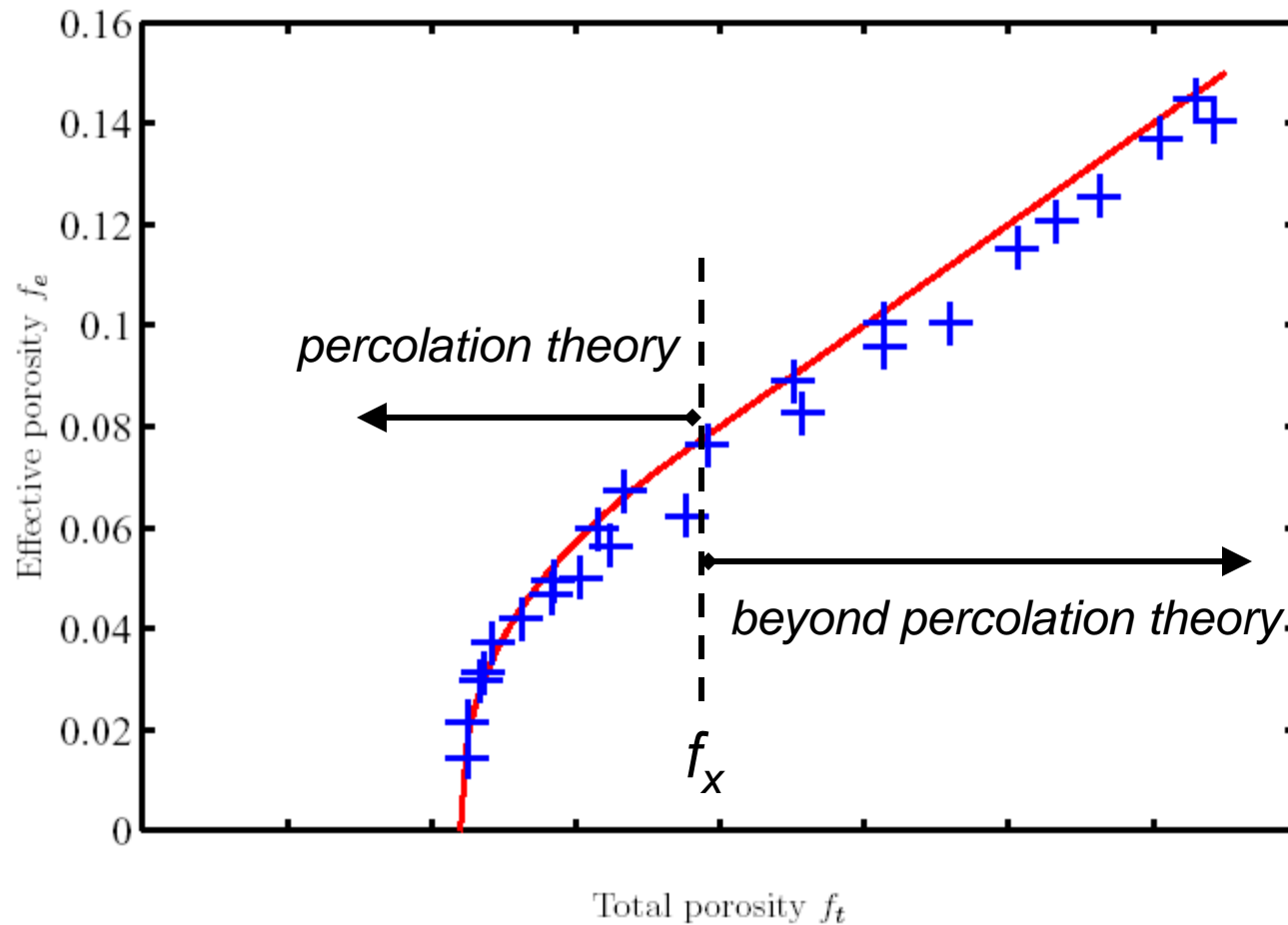
Example: compressed calcite aggregates (Zhang et al., 1994)



$$f_c = 0.044$$
$$f_x = 0.075$$

this model seems to be applicable to *some* porous media

Applicability to sea ice?



need data!

Hence,

crude estimate of the critical porosity of sea ice could be:

$$\text{critical porosity of a 3D system (approx. 0.3)} \times \frac{\text{Non-excluded volume}}{\text{Total volume}} = 0.04 \text{ to } 0.05$$

Anderson & Weeks (1958): brine film separation at porosity at 0.12 to 0.15 (if I remember correctly)

Anderson & Weeks (1958): brine film width 70um. Platelet separation: ?

*(NB: more complicated than this:
talk to Hajo about his micrographs)*

*(NB2: situation in warming ice
might again be different)*

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CFD model – quiescent growth

Continuity (mass conservation)

$$\left[1 - \frac{\rho_s}{\rho_l}\right] \frac{\partial \phi}{\partial t} + \frac{\partial(\phi u)}{\partial x} + \frac{\partial(\phi w)}{\partial z} = 0$$

Darcy's law

Momentum conservation

$$\rho_l \left[\frac{\partial(\phi u)}{\partial t} + \frac{\partial(\phi uu)}{\partial x} + \frac{\partial(\phi uw)}{\partial z} \right] = \mu \left[\frac{\partial^2(\phi u)}{\partial x^2} + \frac{\partial^2(\phi u)}{\partial z^2} \right] - \phi \frac{\partial p}{\partial x} - \phi \frac{\mu}{\Pi_x} \phi u$$

$$\rho_l \left[\frac{\partial(\phi w)}{\partial t} + \frac{\partial(\phi wu)}{\partial x} + \frac{\partial(\phi ww)}{\partial z} \right] = \mu \left[\frac{\partial^2(\phi w)}{\partial x^2} + \frac{\partial^2(\phi w)}{\partial z^2} \right] - \phi \frac{\partial p}{\partial z} + \phi \rho g - \phi \frac{\mu}{\Pi_z} \phi w$$

Energy (heat) conservation

$$\frac{\partial T}{\partial t} + c \rho \left[\frac{\partial(T \phi u)}{\partial x} + \frac{\partial(T \phi w)}{\partial z} \right] = \frac{\partial}{\partial x} \left[\bar{k} \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial z} \left[\bar{k} \frac{\partial T}{\partial z} \right] - [T \Delta(\rho c) + L \rho_s] \frac{\partial \phi}{\partial t}$$

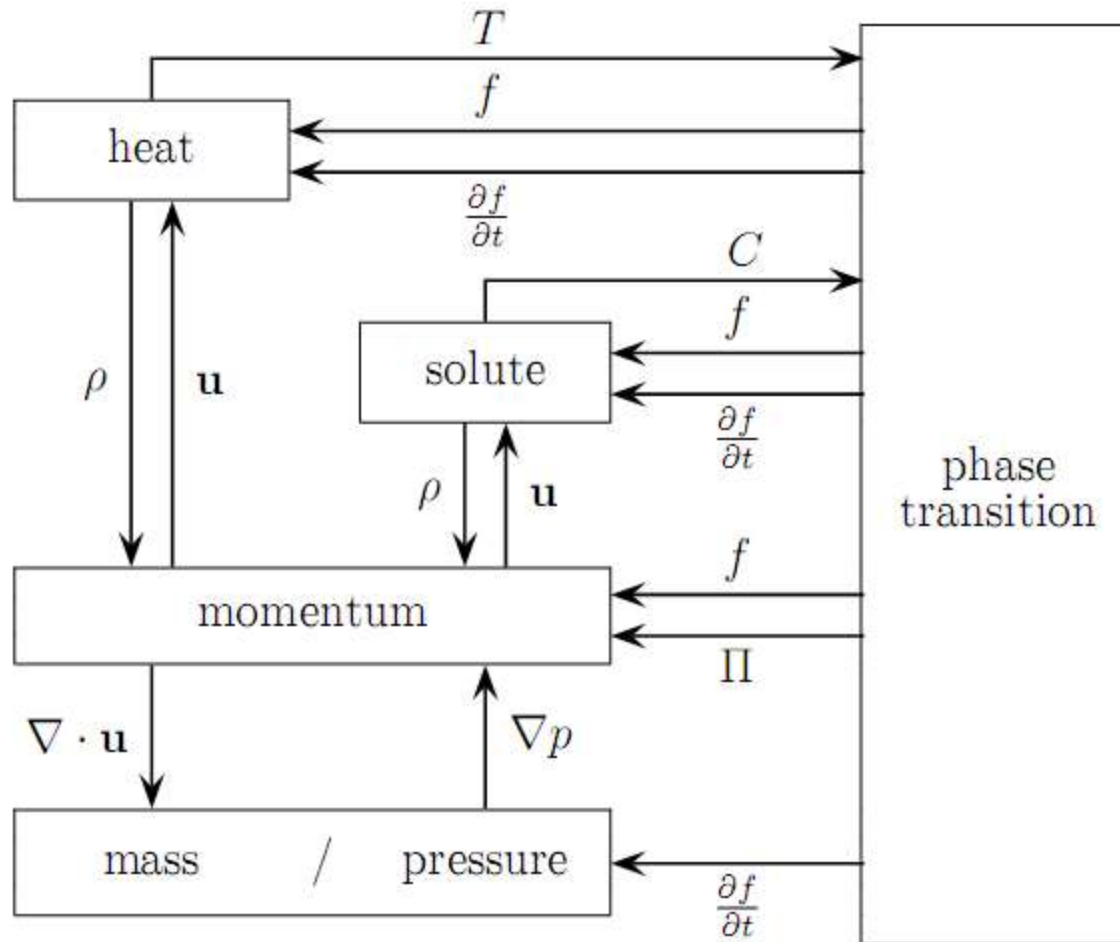
Solute (salt) conservation

$$\phi \frac{\partial C}{\partial t} + \frac{\partial(C \phi u)}{\partial x} + \frac{\partial(C \phi w)}{\partial z} = \frac{\partial}{\partial x} \left[\phi D \frac{\partial C}{\partial x} \right] + \frac{\partial}{\partial z} \left[\phi D \frac{\partial C}{\partial z} \right] - C \frac{\partial \phi}{\partial t}$$

Thermodynamic equilibrium

$$\Delta \phi = (T - T_F) \left[\frac{T \Delta(\rho c) + \rho_s L}{\bar{\rho} c} - \frac{C}{\phi} \left(\frac{\partial T_F}{\partial C} \right)_{\text{at } C} \right]^{-1}$$

Coupled governing equations

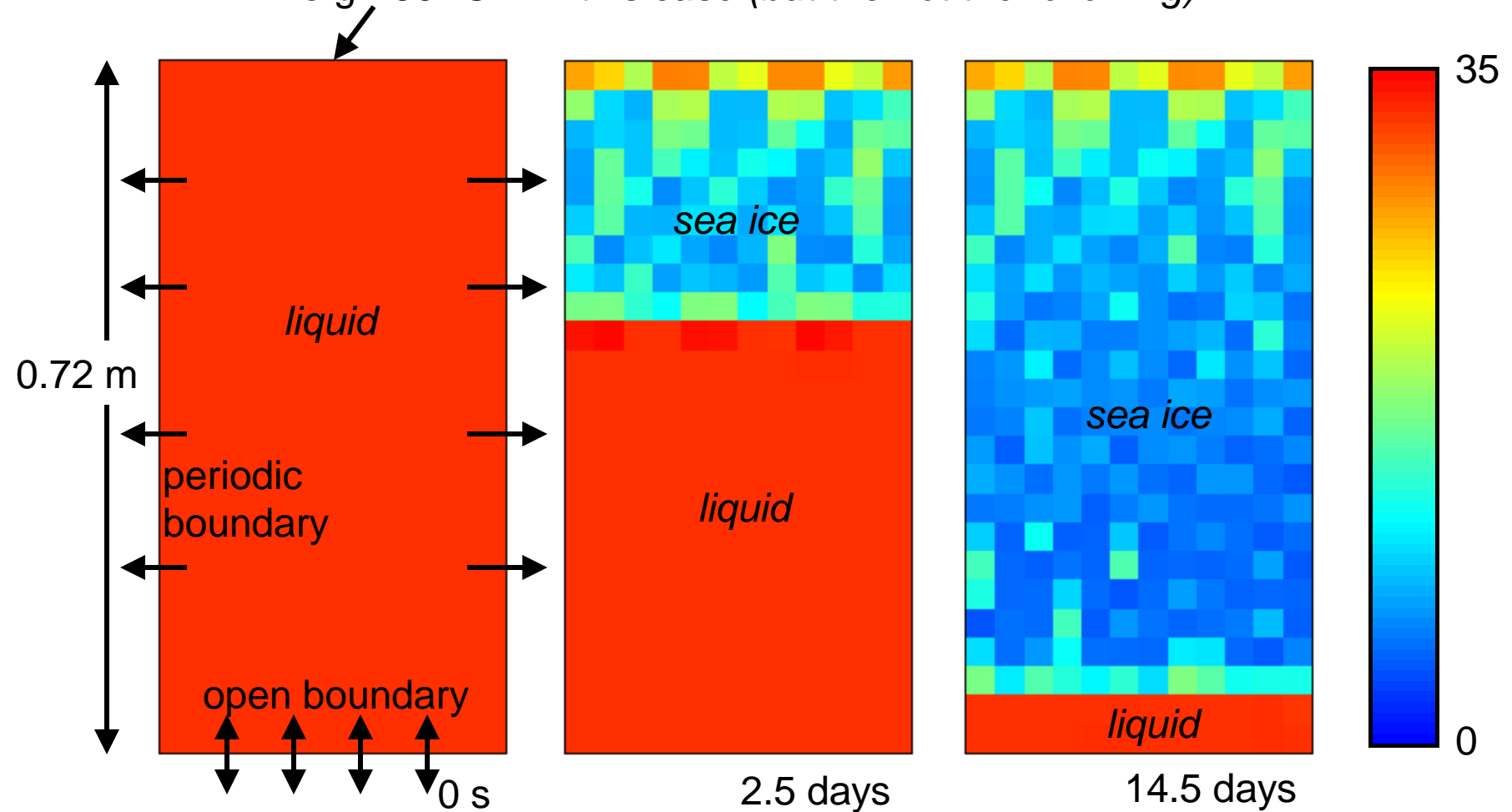


Solved on staggered, rectangular grid w/ multigrid solver

Example:

Development of the bulk salinity (salinity of the melt)

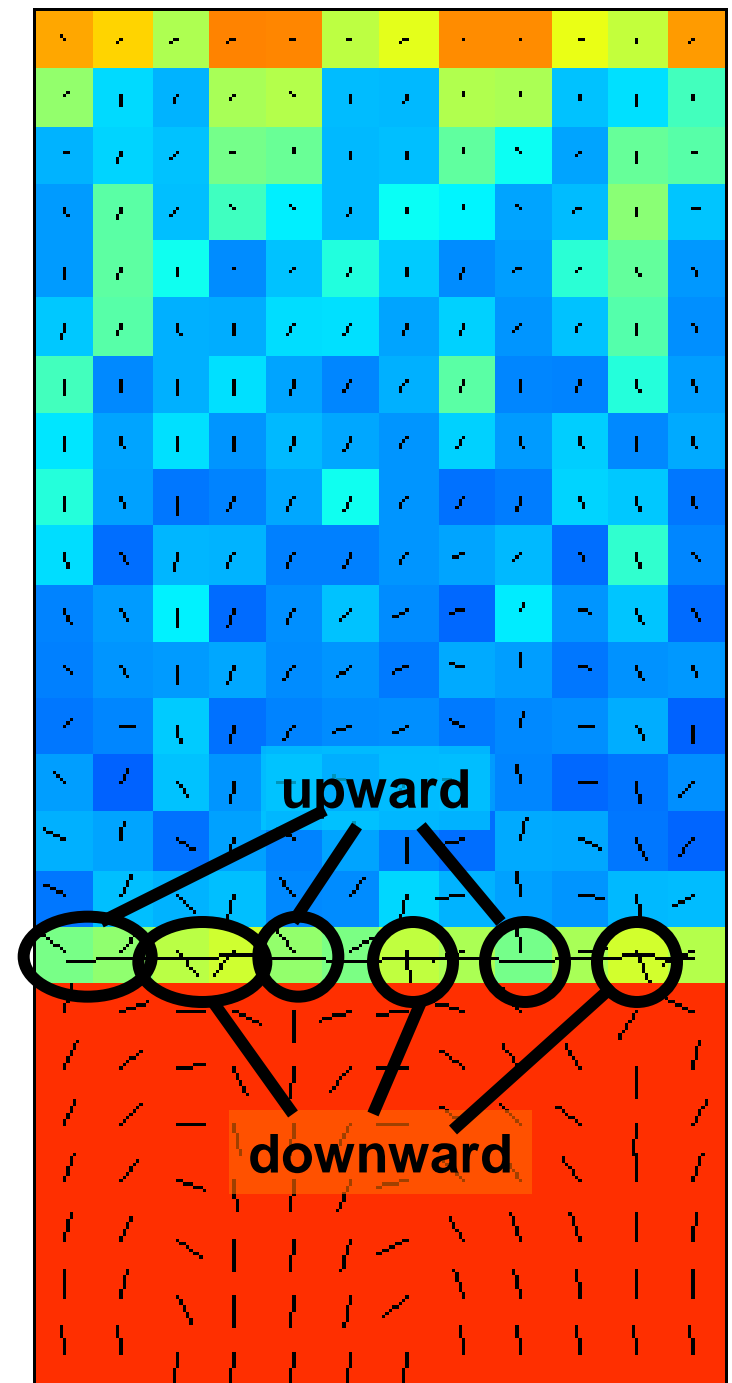
e.g. $-30\text{ }^{\circ}\text{C}$ – *in this case (but the not the following)*



Temperature of brine entering domain: 10 mK above freezing point

Advective ice—ocean interface flux from CFD model

- log turbulent volume flux
(i.e. flux less mean)
at the ice—ocean interface
- plot as function of growth rate



Simulations *in this presentation* use

$$\Pi_z(\phi) = \Pi_x(\phi) = 10^{-8} \text{ m}^2 \phi^3$$

Volume flux,
sea ice bulk salinity,
salinity scatter, and
porosity profile

are sensitive to permeability–porosity relationship.

Ice growth and desalination from the perspective of computational fluid dynamics simulations

As we go smaller to 250um grid size, we find persistent channels with >1 cell width.
Hence, we get a ballpark estimate for brine channel diameters of 0.5 mm.

Flow reversal at the end of the lifetime of a channel.

Counter flow in persistent brine channel.

In spite of thermodynamic equilibrium enforced at 10 ms time step:

lines of constant salinity & isotherms are discontinuous at brine channels

→ non-trivial to estimate temperature and salinity inside convecting brine channels

Features that resemble feeder channels.

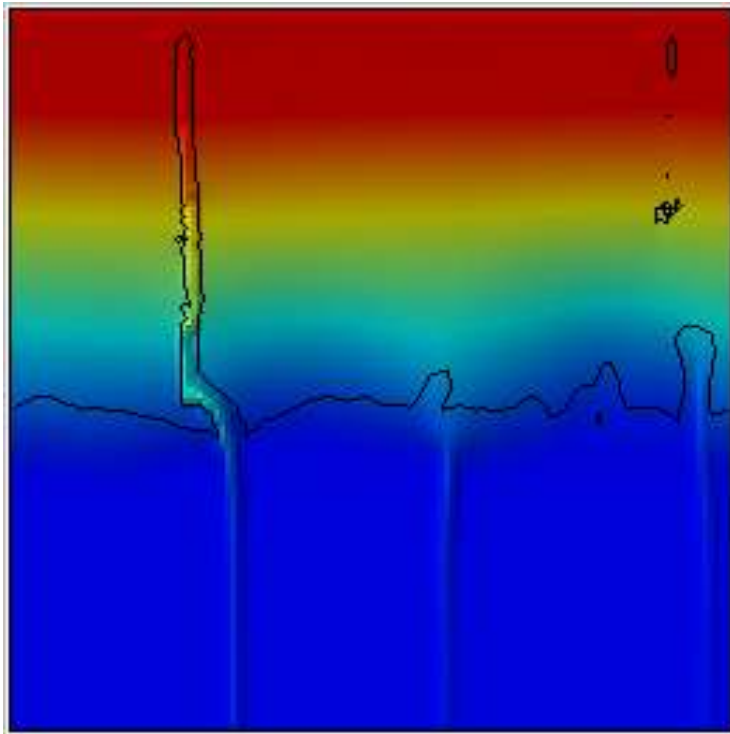
Fluid Dynamics Simulations of Desalination

3 cm x 3 cm domain size

250 μm grid, 60 K/m surface temperature gradient

shaded for contrast

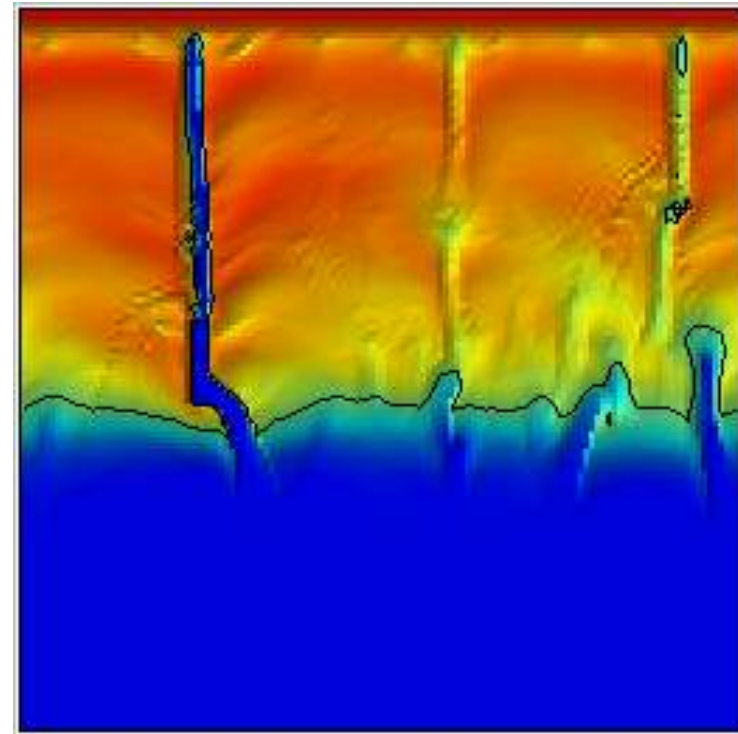
Brine Salinity



50 ppt

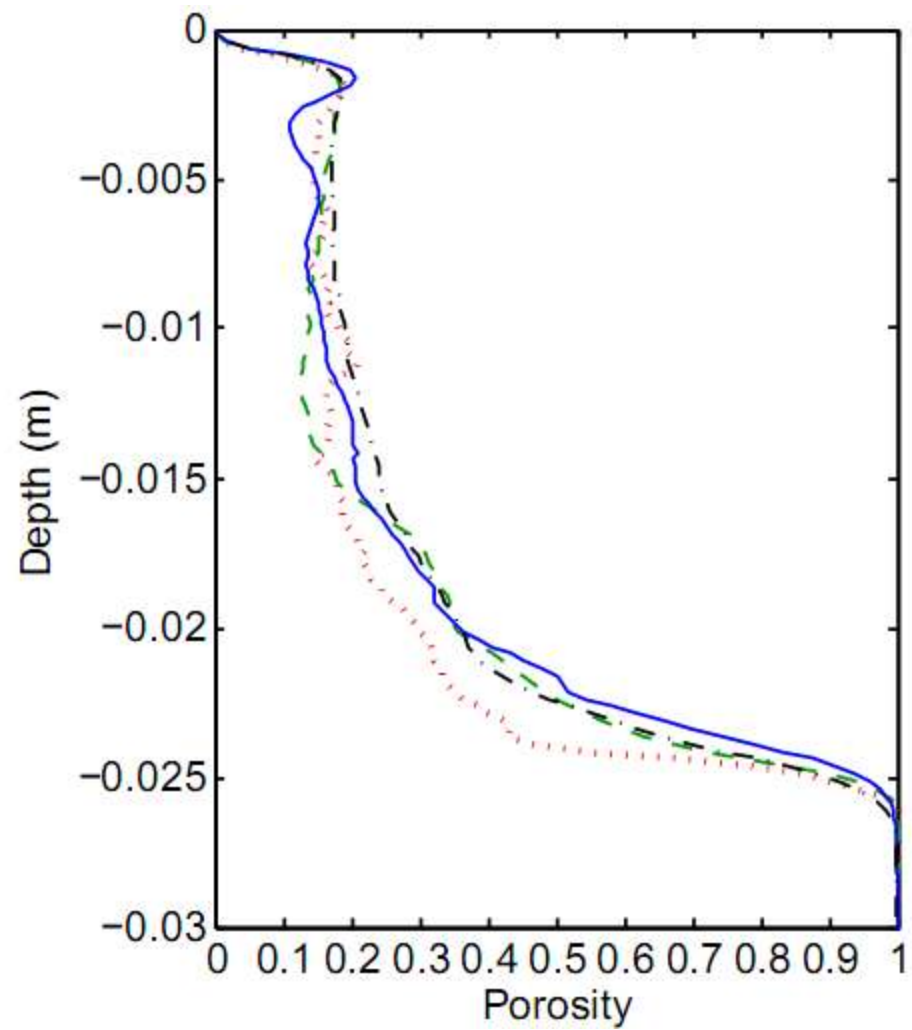
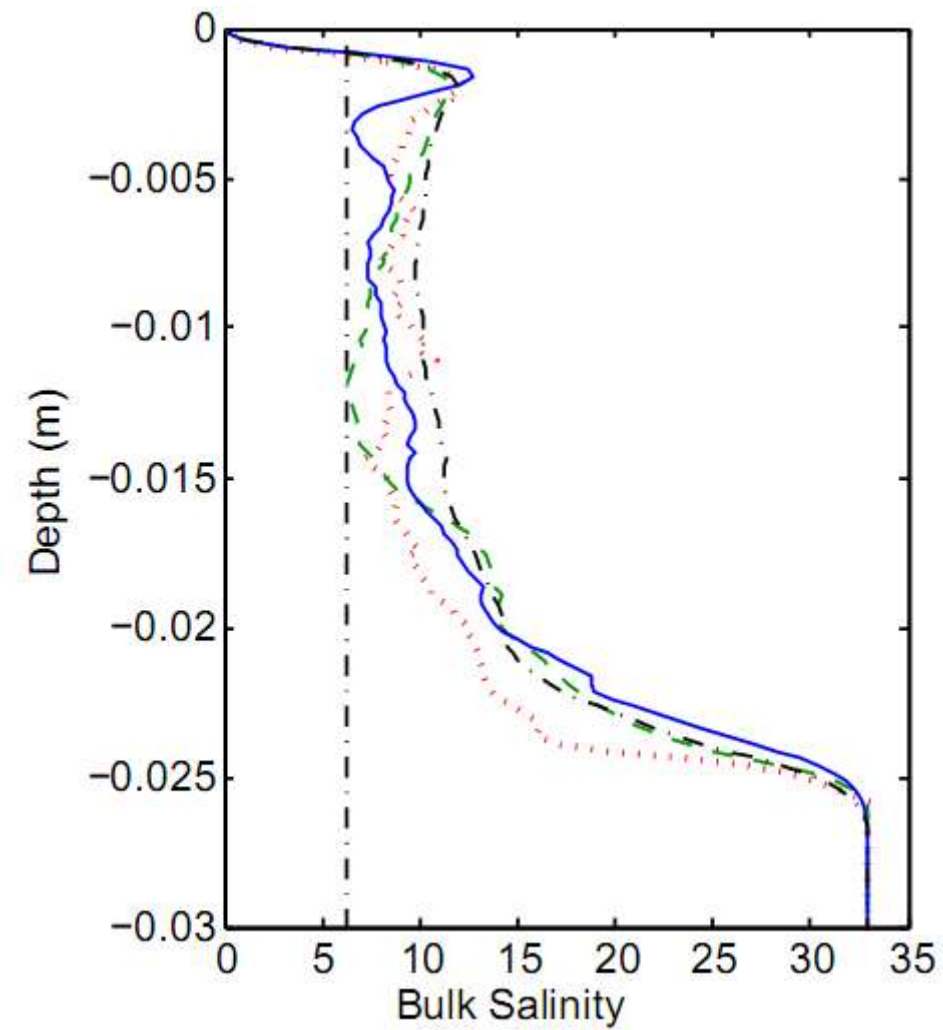
34 ppt

Porosity



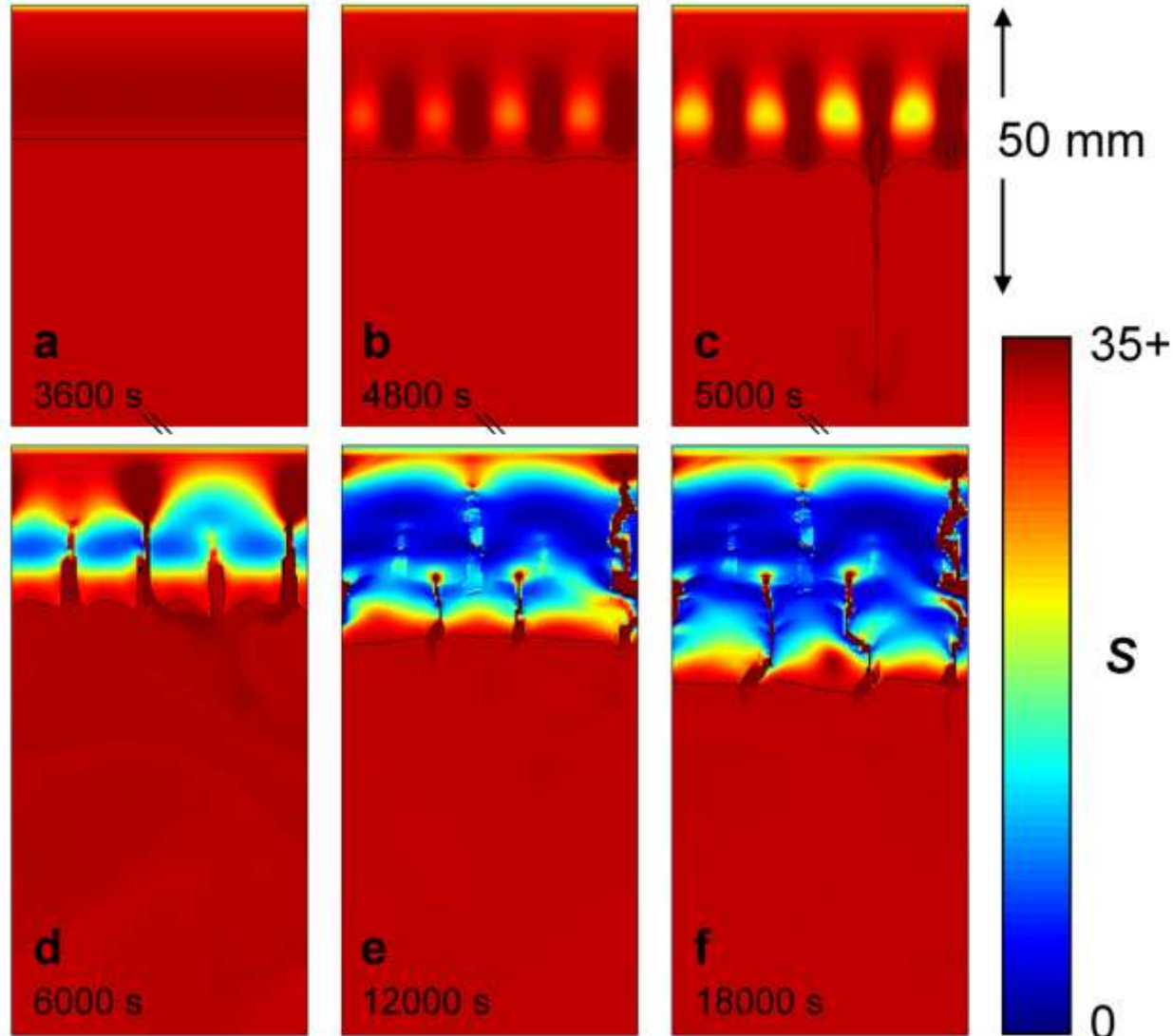
(show animations)

Simulations just shown: blue line



Bulk Salinity – Summay

surface temperature -8 °C, 0.4 mm grid



Seawater salinity: 33

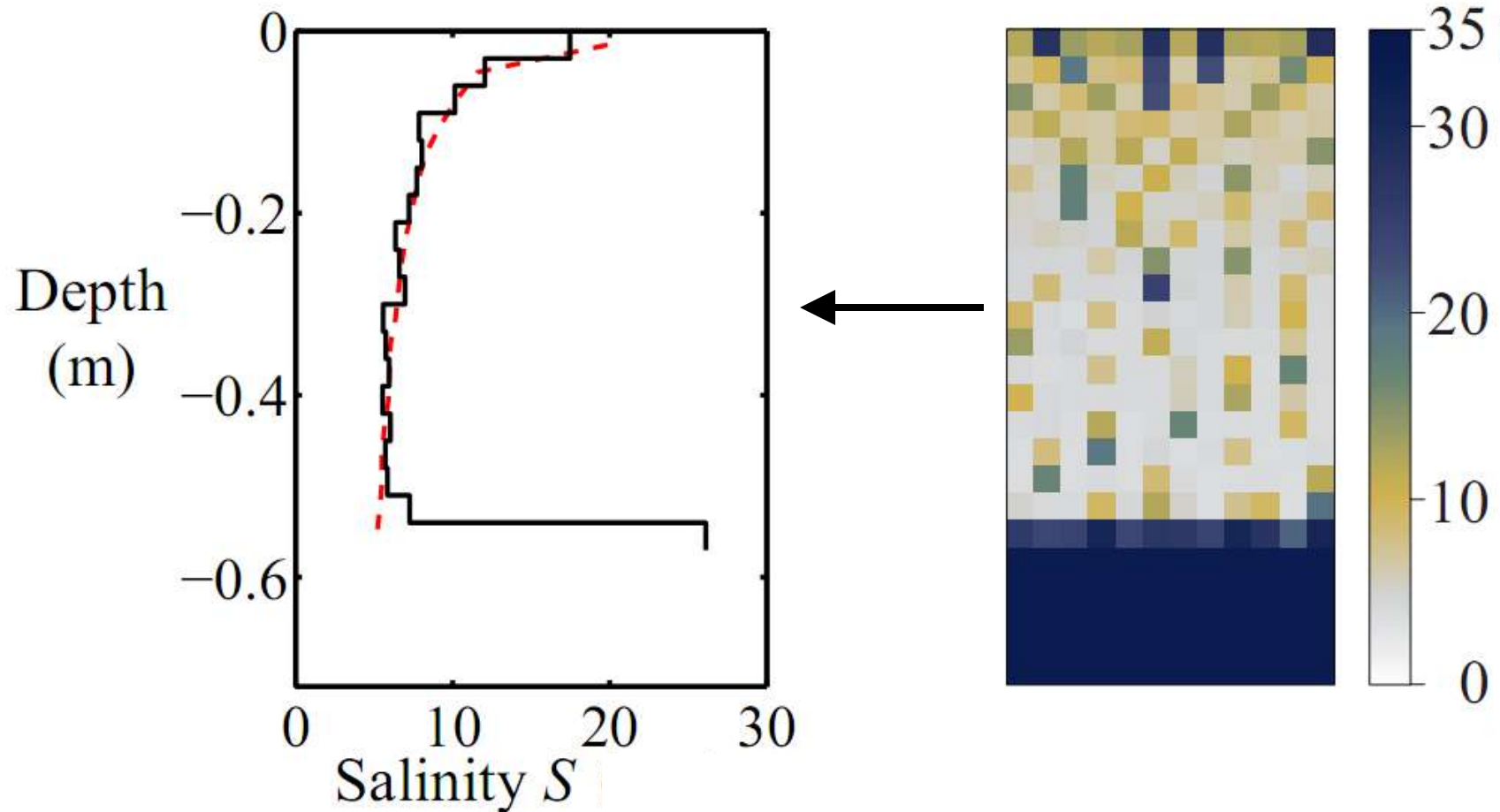
Contour line

traces porosity $\phi = 0.98$.

$$\Pi_z(\phi) = \Pi_x(\phi) = 10^{-9} \text{ m}^2 \phi^2$$

Simulation #60

Averaged salinity profile

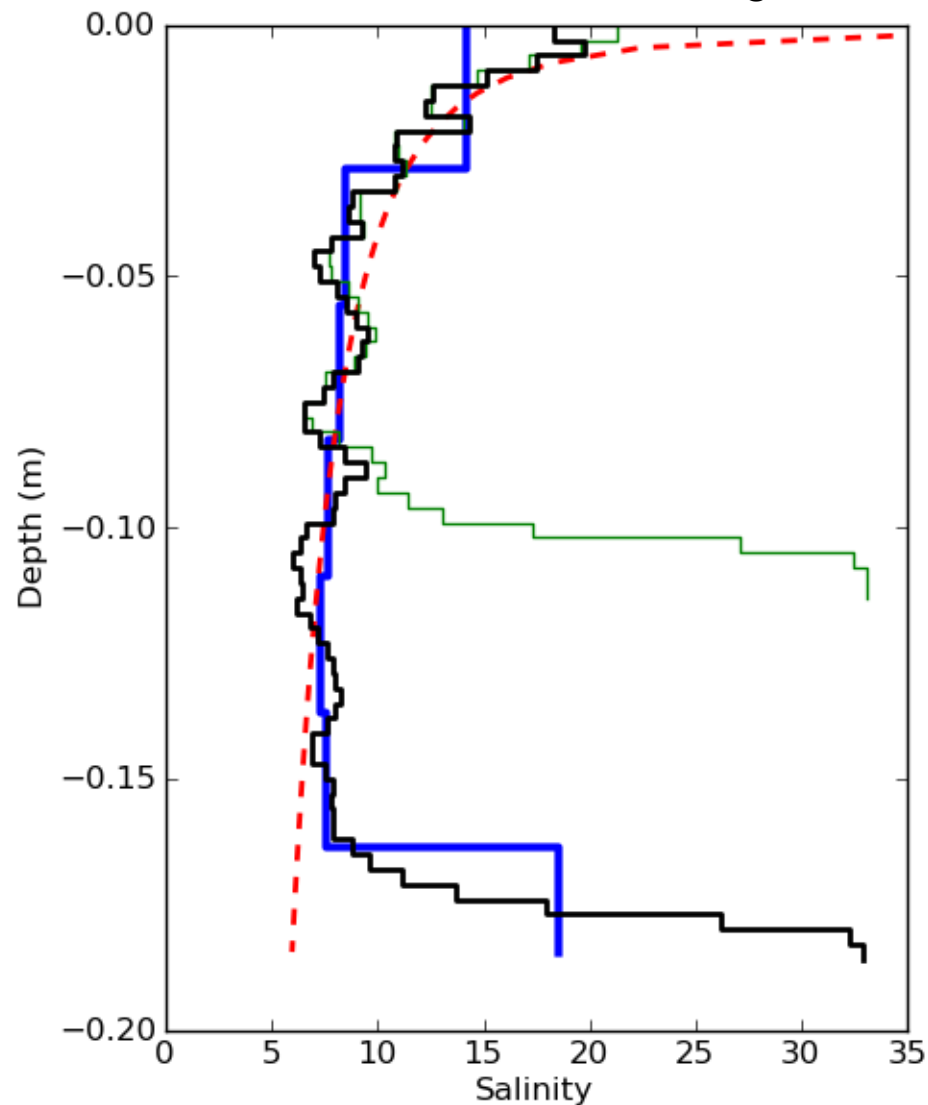


note: slow growth \rightarrow low bulk salinity

Dashed line:
$$\frac{S}{S_0} = 0.14 \left(\frac{v}{1.35 \times 10^{-7} \text{ m s}^{-1}} \right)^{0.33}$$

for $S_0=33$, insignificant ocean heat flux

-10 C surface T, 3 mm grid



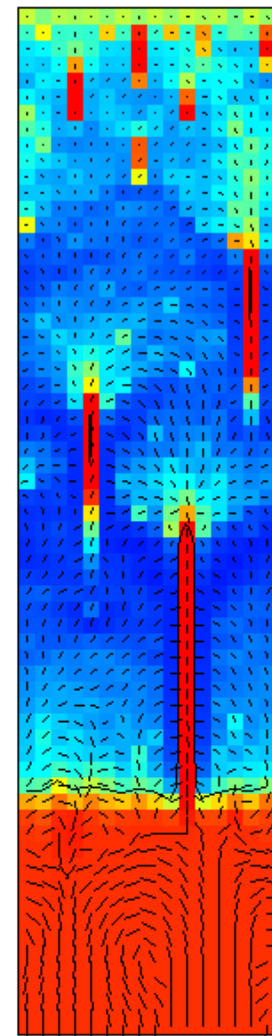
Simulated “stable” salinity follows expectation

except near the ice–ocean interface

5 cm/day

2.5 cm/day

run #39



Measure ice growth and environmental data

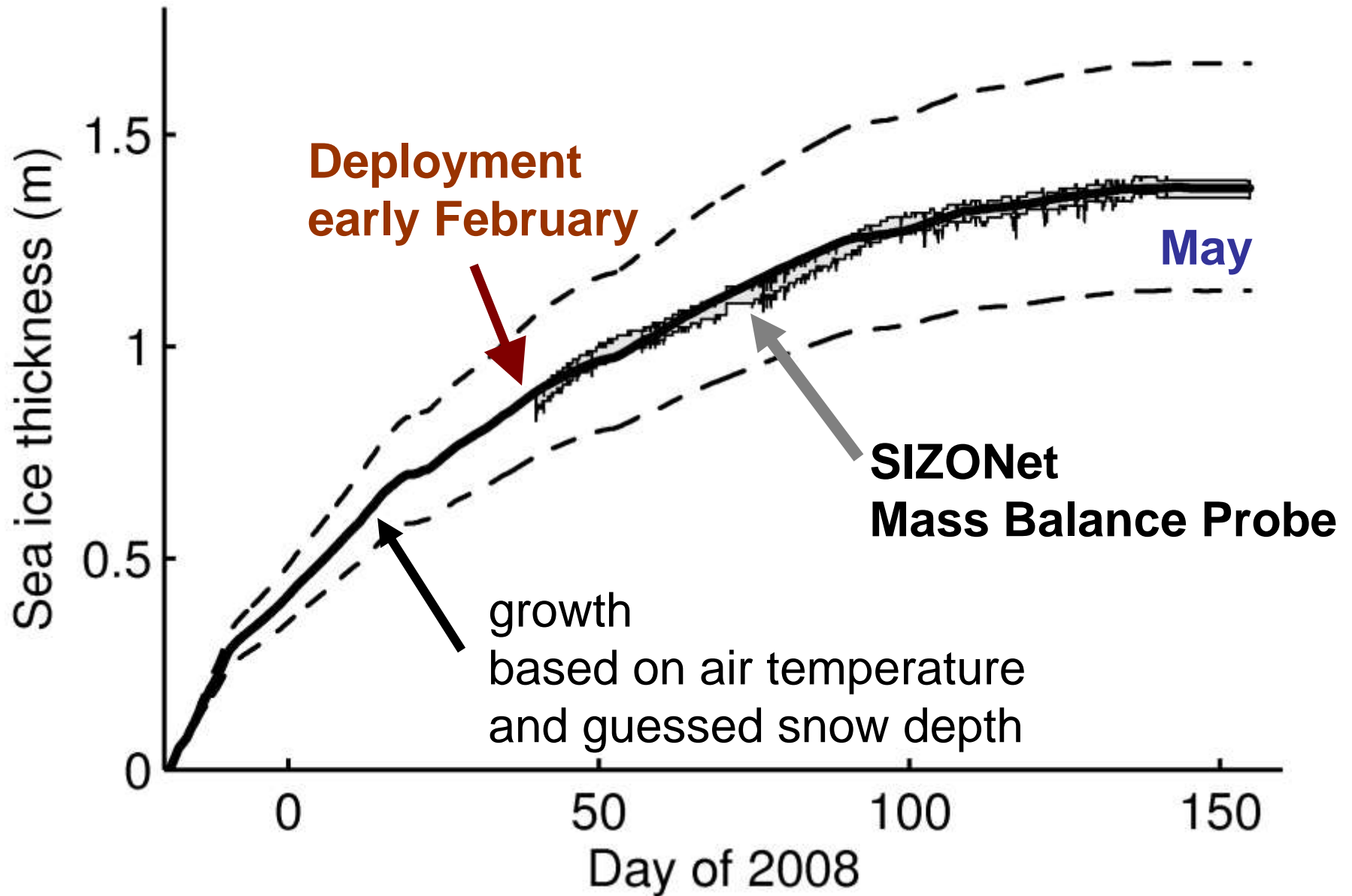
(cf. Hajo's talk)



Barrow, AK, Jan 2009

C. Petrich

Barrow Sea Ice Growth

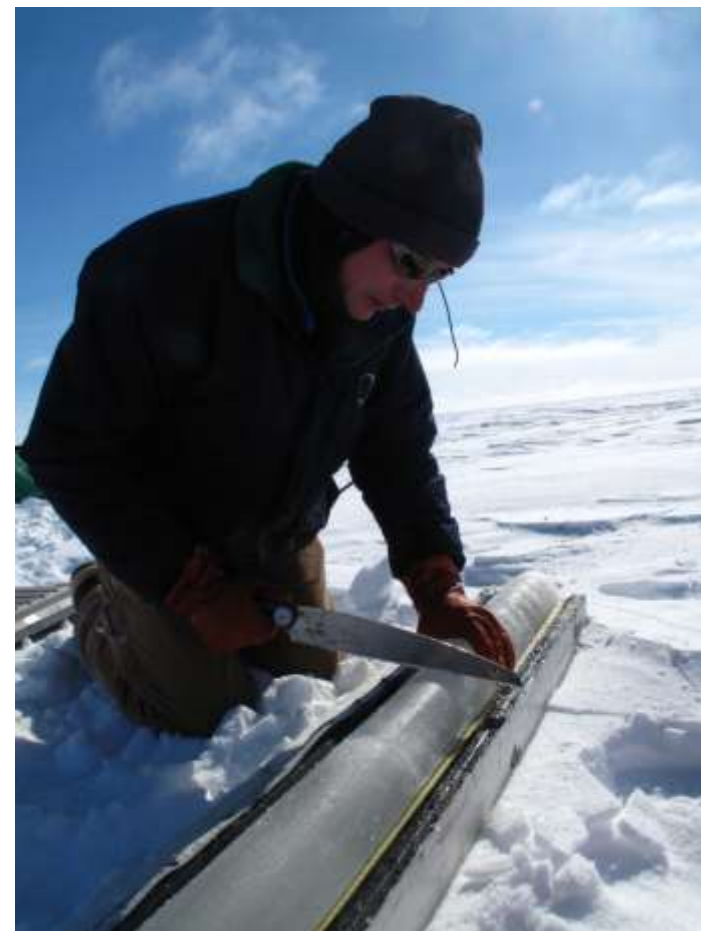


from: Petrich and Eicken in Thomas & Dieckmann, 2nd ed (2010)

Take ice cores



Cut samples

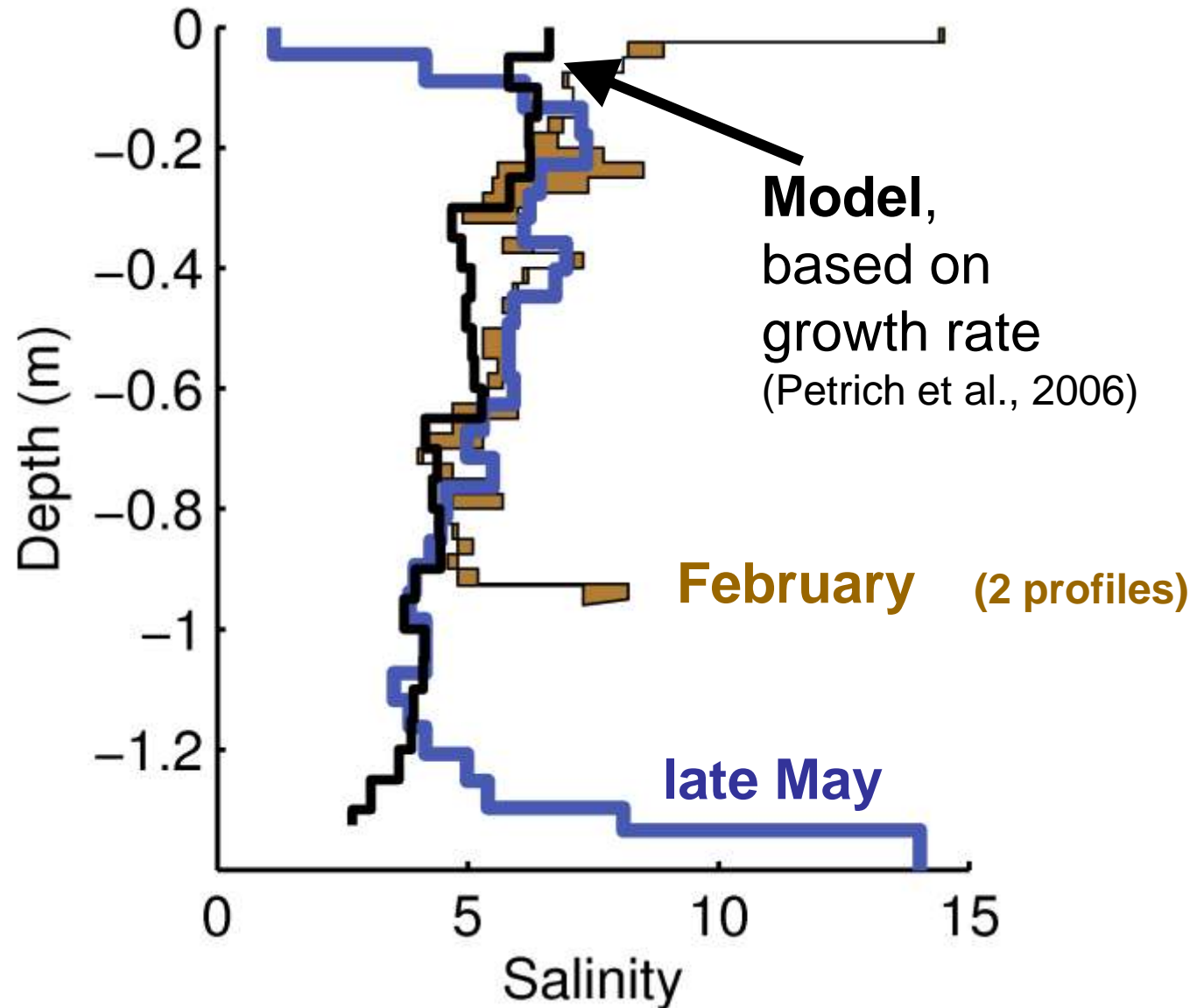


Measure salinity



photo: Polona Rozman

Measured and modeled salinity, Barrow 2008



from: Petrich and Eicken in Thomas & Dieckmann, 2nd ed (2010)